

Feasibility of Measuring the Cosmological Constant Λ and Mass Density Ω using Type Ia Supernovae

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ABSTRACT

We explore the feasibility of resurrecting the apparent magnitude-redshift relation for a “standard candle” to measure the cosmological constant and mass density. We show that type Ia supernovae, if measured with 0.15 mag uncertainty out to a redshift of $z = 1$, may provide a good standard candle or calibrated candle for this purpose. The recent discovery of probable type Ia supernovae in the redshift range $z = 0.3$ to 0.5 (Perlmutter *et al.* 1994a, and 1994b) indicates that the flux of optical photons from these events can be measured this accurately. The 7 distant supernovae discovered to date do not by themselves distinguish between different cosmological models, however the further discovery of about 50 type Ia supernovae at redshifts in the range $0.5 \leq z \leq 1.0$ could strongly constrain the allowed range of these parameters. We estimate that the follow-up photometry necessary for this measurement would be on the order of 20 – 70 hours of time on a 10-meter class telescope at a site with good seeing.

Subject headings: cosmology: observations, cosmological constant—supernovae

1. Introduction

Recent attention to the problem of measuring or bounding the cosmological constant Λ has yielded inconclusive results. The review article of Carroll, Press, & Turner (1992) surveyed the observational status of the cosmological constant based on (1) the existence of high-redshift objects, (2) the ages of globular clusters and cosmic nuclear chronometry, (3) galaxy counts as a function of redshift or apparent magnitude, (4) dynamical tests (clustering and structure formation), (5) quasar absorption line statistics, (6) gravitational lensing counts and statistics, and (7) the astrophysics of distant objects. The conclusion of this exhaustive survey was that the current best “observationally secure” bounds on the cosmological constant are $-7 < \Lambda/(3H_0^2) < 2$, leaving a wide range of possible cosmological models to choose from. In fact, we still do not know if we live in an infinite universe that will expand infinitely or a finite universe that at some point will halt its expansion and recollapse. In this paper we explore the feasibility of resurrecting the apparent magnitude-redshift relation for a “standard candle” as an eighth method to add to this arsenal of measurement techniques.

The early work on the implications of cosmological models on the apparent magnitude-redshift (m - z) relation of a standard candle, the first-ranked cluster galaxies, did consider the possibility of a non-zero cosmological constant (e.g., Solheim 1966, Stabell & Refsdal 1966). As the difficulties of studying evolutionary effects for these galaxies became clear, the range of cosmological models considered narrowed to just those with a vanishing cosmological constant (e.g., Peach 1970). The equations of galaxy evolution and the deceleration parameter q_0 (or equivalently the mass density of the universe Ω_M) were considered complications enough in these m - z studies. The most recent work has generally been considered to be more a study of evolution than a measurement of q_0 or Ω_M (for a review see Sandage 1988). In the past few years new evidence has been put forward

suggesting that a group of type Ia supernovae (SNe Ia) can be identified that are excellent standard candles or calibrated candles. There is reason to believe that evolution effects should be much less significant for SNe Ia than for first-ranked cluster galaxies and that even if present such effects may be distinguishable on an event by event basis. The past few years have also seen the start of searches for distant SNe, resulting in the discovery and study of 7 SNe Ia at redshifts in the range $z = 0.3$ to 0.5 (Norgaard-Nielsen *et al.* 1989; Perlmutter *et al.* 1994a, 1994b). This is clearly an opportune time to reconsider the use of standard candles to measure Λ .

In this paper we first review the current understanding of the usefulness of a sub-group of SNe Ia as standard candles, and the possibility of further “calibrating” these candles using lightcurve decay-time or shape. We then discuss the use of standard candles to measure the cosmological constant and mass density. Some of the earliest papers that treated the $\Lambda \neq 0$ case pointed out that the magnitude–redshift measurement was insensitive to q_0 at certain redshifts while still sensitive to Ω_M (e.g. Refsdal, Stabell, & Lange 1967). We propose to take advantage of this redshift dependence to measure Ω_M and Λ simultaneously. The special case of a “flat” universe, as implied by the inflationary theories of the universe, is discussed separately. We then draw conclusions about the observational requirements and hence the feasibility of a new measurement of Λ and Ω_M using SNe Ia.

2. Type Ia supernovae

There is much evidence indicating that a distinguishable majority of type Ia supernovae are likely to be good standard candles. The problem of estimating the intrinsic dispersion of SNe Ia, however, has been clouded by the inclusion of supernovae with peculiar spectra or lightcurves, supernovae showing clear evidence of host-galaxy extinction, and supernovae that had very large uncertainties on their photometry measurements. For a subsample of

well-measured “local” SNe Ia that do not have peculiar spectra or lightcurves and do not show clear evidence of extinction, the *observed* dispersion is $\sigma_V = \sigma_B = 0.3$ magnitude in both the V and B bands (Vaughan *et al.* 1994). This dispersion of these “normal” SNe Ia is completely accounted for by measurement errors (most of this scatter is probably due to the relative-distance measurement error) and thus the *intrinsic* dispersion is likely to be smaller than this. Sandage & Tammann (1993) use Monte Carlo modeling of Malmquist bias to argue that the largest intrinsic dispersion for type Ia supernovae which is compatible with the observed selection effects for nearby supernovae is $\sigma_{M_V}^{intrinsic} \approx 0.2$ mag.

Vaughan *et al.* proposed that their criteria for “normal” SNe Ia be tested on an independent set of SNe to confirm the small observed observed dispersions σ_V and σ_B . Hamuy *et al.* (1994) presented such an independent, new set of SNe Ia, including both “normals” and “peculiar,” with smaller measurement errors. In particular, the relative-distance measurement error was smaller, because this set of SNe was discovered at redshifts $z \approx 0.01$ to 0.1 where the peculiar velocities are negligible with respect to the Hubble flow. Selecting just the “normal” SNe Ia from this set, using the criteria of Vaughan *et al.*, results in an even narrower observed dispersion of $\sigma_V = 0.23$ mag in the V band and $\sigma_B = 0.25$ mag in the B band (Vaughan, Branch, & Perlmutter 1994).

Hamuy *et al.* (1994) and Riess, Press, & Kirshner (1994) also reported a correlation of lightcurve decay-time or lightcurve shape with peak absolute magnitude for this set of SNe Ia. (Note that this correlation would not be easily found in the earlier set of SNe Ia with larger measurement errors, although Phillips 1993 did report such a relation for a small sample of well-measured peculiar and normal SNe Ia.) Using this correlation to provide a “calibration” of the SN Ia standard candle may make it possible to include peculiar SNe Ia in distance measurements. The correlation also appears to hold within the “normal” SN Ia subset, allowing even this subset’s already narrow dispersion to be further reduced

after calibration, yielding σ_V as low as 0.12 (Hamuy *et al.* 1994) or 0.21 (Riess, Press, & Kirshner 1994). Although this calibrated σ_V would imply a still smaller intrinsic dispersion, for this paper we will take the intrinsic dispersion to be the “uncalibrated” value, which is bounded by the observed dispersions to be $\sigma^{intrinsic} < 0.25$ in the V or the B band. This is a conservative value, given that the *observed* dispersions quoted in Vaughan, Branch, & Perlmutter (1994) are less than or equal to this.

If SNe Ia are to be more useful as cosmological standard candles than the first-ranked cluster galaxies have been, they either must not evolve in absolute magnitude or this evolution must be easily detected and characterized. There are at least two reasons suggesting that SN Ia standard candles should not founder on the evolution problem:

(1) Unlike first-ranked cluster galaxies, SNe Ia are dynamic events that display their internal composition and physical state through the many spectral lines that appear, shift in velocity, and disappear, and also through the photometric lightcurves in various wavelength bands. It is possible to observe each individual SN Ia, match its spectra over time and lightcurves against those of nearby SNe Ia, and check for subtle changes from the range of normal SNe Ia. These changes are very likely to be *more* sensitive to the details of the precursor star and environment than the peak absolute magnitude is, and thus can provide “early warning” before there are differences large enough to affect the absolute magnitude significantly. For example, the lightcurve decay-time or shape and the spectral absorption line velocities both appear to be sensitive indicators of explosion strength.

(2) SNe Ia have been discovered in a wide range of nearby galaxy types. This variation in host galaxy environment can be used as a surrogate for the variation that would be expected due to evolution. This has been done, for example, by Branch & van den Bergh (1993), who suggest that Si II absorption line velocity may be correlated with host galaxy type. Branch & van den Bergh did not see a correlation with absolute magnitude in this

case, but such studies of nearby supernovae can in principle detect, and provide tests for, evolution of absolute magnitudes. Ideally these tests would make it possible to distinguish degrees of evolution on a supernova-by-supernova basis.

Even if the SNe Ia themselves do not evolve, it is possible that the host galaxy dust may evolve, thus changing the apparent magnitude with redshift. Although very careful color photometry should provide checks for this effect, it is probably easier to compare SNe Ia in different galaxy types (both nearby and distant), once again using these types as surrogates, this time for evolution of host galaxy dust. So far there does not appear to be such an effect for a range of nearby galaxy types.

These evolution tests will provide the underlying proof of SNe Ia as standard candles or calibrated candles, and could of course someday find some SNe Ia exhibiting evolution effects that cannot be easily corrected. It is important to re-emphasize, however, that SNe Ia are unusual standard candles in having such tests available on an individual basis: each SN Ia can be accepted or rejected by itself.

3. Constraining the parameters by standard candle luminosity distance

For an object of known absolute magnitude M , a measurement of apparent magnitude m at a given redshift is sensitive to the universal parameters Ω_M and $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$ through the luminosity distance D_L :

$$m = M + 5 \log[D_L(z; \Omega_M, \Omega_\Lambda)] + K + 25, \quad (1)$$

where the K -correction in the equation appears because the emitted and detected photons from the receding object have different wavelengths. The dependence of D_L on Ω_M is different from the dependence on Ω_Λ , entering with different powers of z :

$$D_L(z; \Omega_M, \Omega_\Lambda) = \frac{(1+z)}{H_0 \sqrt{|\kappa|}} \mathcal{S} \left(\sqrt{|\kappa|} \int_0^z \left[(1+z')^2 (1 + \Omega_M z') - z' (2 + z') \Omega_\Lambda \right]^{-\frac{1}{2}} dz' \right), \quad (2)$$

where, for $\Omega_M + \Omega_\Lambda < 1$, $\mathcal{S}(x)$ is defined as $\sin(x)$ and $\kappa = 1 - \Omega_M - \Omega_\Lambda$; for $\Omega_M + \Omega_\Lambda > 1$, $\mathcal{S}(x) = \sinh(x)$ and κ as above; and for $\Omega_M + \Omega_\Lambda = 1$, $\mathcal{S}(x) = x$ and $\kappa = 1$.

Using Equations (1) and (2) we can predict the apparent magnitude of a standard candle measured at a given redshift for any pair of values of Ω_M and Ω_Λ . Note that the value of the Hubble parameter drops out of the equations as it appears both in the expression for the luminosity distance and in the determination of the absolute magnitude of the standard candle based on nearby apparent magnitude–redshift measurements. Figure 1 shows the contours of constant apparent magnitude in the R-band on the Ω_Λ -versus- Ω_M plane, for the cases of $z = 0.5$ and $z = 1$, where we have taken the absolute luminosity of type Ia supernovae to be $M_B = -18.86 \pm 0.06 + 5 \log(H_0/75)$ (Branch & Miller 1993, Vaughan *et al.* 1994).

When an actual apparent magnitude measurement is made of a standard candle, for example at $z = 0.5$, the range of possible values of Ω_M and Ω_Λ are narrowed to a single contour line on Figure 1 (dashed lines for $z = 0.5$). Given some uncertainty in the apparent magnitude measurement, the allowed range of Ω_M and Ω_Λ is given by a strip between two contour lines. Two such measurements for standard candles at different redshifts (for example $z = 0.5$ and $z = 1$) can define two strips that cross in a more narrowly constrained “allowed” region, shown as a shaded rhombus in Figure 1. The darker shaded region in the plot corresponds to the result of measurements with 0.05 mag uncertainty in a flat universe with vanishing cosmological constant, while the faint region allows for a 0.10 mag uncertainty at $z = 1$. Note that the one-standard-deviation error region is limited by an ellipse rather than the rhombus in Figure 1. To simplify this figure and the following two figures, we have not drawn the 1σ error ellipse.

In the case where a standard candle is measured at $z = 0.5$ and $z = 1$, Figure 2 shows the allowed regions for Ω_Λ and Ω_M for a set of three example universes *superposed* on the

same graph (i.e. the actual measurements would result in only one of the shaded regions A, B, or C). Note that on this graph, very large positive values of Ω_Λ are ruled out because they would imply a “bouncing” (no Big Bang) universe, as discussed in Carroll, Press, & Turner (1992). Also extremely large values of Ω_M combined with negative Ω_Λ are ruled out because they would imply a universe younger than the oldest heavy elements, which have been dated to be 9.6 Gyr (Schramm 1990). The shaded regions correspond to hypothetical results in a universe with parameters ($\Omega_\Lambda=0.5$, $\Omega_M=0.5$) for A, ($\Omega_\Lambda=0.0$, $\Omega_M=1.0$) for B, and ($\Omega_\Lambda=-0.5$, $\Omega_M=1.5$) for C. These examples all correspond to flat universes, but with different contributions from matter and cosmological constant. Similarly, figure 3 shows how this method would distinguish the case D ($\Omega_\Lambda=0.0$, $\Omega_M=0.2$) from E ($\Omega_\Lambda=0.8$, $\Omega_M=0.2$).

In practice, more than two apparent magnitude measurements at two redshifts would be used for this measurement. A global fit of Equations (1) and (2) to the measurements would then yield best-fit contours on the Ω_Λ -versus- Ω_M plane. Figures 1 through 3, however, give a direct understanding of how good the measurement errors need be to constrain Ω_Λ and Ω_M : the accuracy of the magnitude measurements translates into a region in the Ω_Λ versus Ω_M parameter space approximately as $(\Delta\Omega_\Lambda \times \Delta\Omega_M) \propto (\sigma_m^{z=0.5} \times \sigma_m^{z=1.0})$. Note that these magnitude errors are the combined error of the apparent magnitude measurement at redshift z , the absolute magnitude estimate for the standard candle used, and the intrinsic dispersion of SNe Ia. We see from the figures that a combined measurement error of $\sigma_m \leq 0.05$ mag significantly constrains Ω_Λ and Ω_M .

In this paper we assume that the photometric measurements are going to be sufficiently precise that the intrinsic dispersion of SNe Ia dominates, $\sigma^{intrinsic} < 0.25$ mag (in section 5 we discuss the observational requirements to achieve this photometric accuracy). In order to make the ± 0.05 mag measurement at $z = 0.5$ and $z = 1$ shown in figures 1 through 3, we thus must have a sample of at least 25 supernovae at each redshift.

4. Ω_Λ in the flat universe case

An important special case to consider is the “flat” universe predicted by the inflationary theories, where the total energy density of the universe $\Omega_T \equiv \Omega_\Lambda + \Omega_M = 1$. [The other special case with $\Lambda = 0$ has been discussed in Perlmutter *et al.* (1994a).] In a flat universe, the apparent magnitude of a standard candle as a function of redshift is extremely sensitive to Ω_Λ . Figure 4 shows the theoretical curves for the luminosity distance as a function of redshift for flat universes. A measurement of the apparent magnitude of a standard candle at $z = 1$ would strongly constrain the cosmological constant and thus test inflationary models. As an example, for the case in which Ω_T is dominated by Ω_Λ , it could be measured with $\sim 10\%$ accuracy even with σ_m as large as 0.25 mag. The ratio of photon flux for the Ω_M -dominated versus the Ω_Λ -dominated case is about a factor of three for a standard candle at $z = 1$.

At redshift $z = 0.458$, where the most distant type Ia supernova was found, the total measurement error, $\sigma_m \approx 0.3$ mag (including the uncertainty in the photometry, $\sigma^{\text{photometry}} \approx 0.15$ mag, as well as the uncertainties in the K -correction and the intrinsic dispersion of type Ia SNe), yielded a 1σ allowed interval of $-0.2 < \Omega_\Lambda < 0.9$ for $\Omega_T = 1$. This allowed interval is shown by the data point and outer error bar in Figure 4. Note that this particular supernova did not have the color measurements that would make it possible to distinguish host galaxy extinction or a peculiar supernova, and therefore this provides only a demonstration data point.

5. Observing requirements

The analysis of the photometry of SN1992bi showed that one can measure the apparent R magnitude at peak of a supernova at $z = 0.458$ with a photometric uncertainty $\sigma^{\text{photometry}} \approx 0.15$ mag (Perlmutter *et al.* 1994a). Using a 2.5 meter telescope and a “thick”

CCD (peak quantum efficiency $\sim 43\%$ at 650 nm), a total of 135 minutes of exposures were required, 90 minutes distributed over the four months near peak and a reference image of 45 minutes one year after peak. The average seeing was approximately 1.5 arcsec. For SN1992bi, the uncertainty at peak relative to the reference image was only 0.06 mag, and the error on the reference image photometry of the host galaxy dominated. Clearly, the longest single exposure should be the one of the reference image of the host galaxy after the SN has faded. In order to take advantage of the further magnitude calibration from lightcurve decay-time or lightcurve shape, this series of observations must begin before maximum light; the search technique of Perlmutter *et al.* (1994a, 1994b) makes this possible on a systematic basis.

Scaling to a 10-meter class telescope, at a site such as Mauna Kea with 0.75 arcsec median seeing and with a thinned CCD, we estimate that the uncertainty in apparent magnitude of distant supernovae at $z = 0.5$ (~ 0.2 mag fainter) can be kept below $\sigma^{\text{photometry}} = 0.15$ magnitudes with 1.5ϵ minutes of photometric measurements, where ϵ accounts for the scaling factors: $\epsilon = (\text{seeing}/0.75'')^2 (10\text{m/aperture})^2$. The photometric uncertainty is dominated by the sky background at these high redshifts, typically more than 4 magnitudes brighter than the counting rate from the SN and the host galaxy. Mauna Kea and La Palma, where SN 1992bi was observed, have essentially the same sky brightness, but at a different site exposure time would scale with sky, too, as $10^{0.8(\text{sky}_1 - \text{sky}_2)}$.

Observing $N = 25$ supernovae at $z = 0.5$ would require less than 1ϵ hour of 10-meter telescope photometry time. The overall measurement uncertainty would then be $\sigma_m = N^{-1/2} [(\sigma^{\text{intrinsic}})^2 + (\sigma^{\text{photometry}})^2]^{1/2} \leq 0.05$ mag, for $\sigma^{\text{intrinsic}} < 0.25$ mag, and neglecting the much smaller error in the mean SN Ia absolute magnitude. This is the value of σ_m discussed in Section 3 and shown in the dark-shaded regions of Figures 1 to 3.

For a type Ia supernova at $z = 1$, $5 \log D_L$ is about 2 magnitudes fainter than for

$z = 0.5$ (see Figure 4 for the effect of different cosmologies on this distance modulus). Although the choice of the R filter is well suited for the $z = 0.5$ supernovae, the I filter is more appropriate for $z > 0.85$, because the rest-frame flux from type Ia supernovae falls rather steeply below ~ 300 nm. The sky is approximately 0.8 magnitudes brighter in the I -band, but the difference of zeropoints between the I and R band is -0.8 magnitudes, so there are roughly the same number of sky background *photons per second* in both R and I in spite of the difference in magnitudes (Massey et al. 1995). Taking into account a reduction of the quantum efficiency by a factor of ~ 2 above 800 nm, it would take approximately 2ϵ hours of observing time per supernova at $z = 1$ to obtain $\sigma^{\text{photometry}} \leq 0.15$ mag uncertainty in the apparent magnitude.

To achieve an overall measurement uncertainty of $\sigma_m \leq 0.05$ mag would then require 34 SN Ia, or $\sim 70\epsilon$ hours of 10-meter photometry time. Alternatively, $\sigma_m \leq 0.1$ mag could be achieved with only 9 SN Ia observed at $z = 1$, requiring $\sim 18 \epsilon$ observing hours. Note that a $\sigma_m = 0.1$ mag uncertainty at $z = 1$ still yields quite useful bounds on the Ω_M versus Ω_Λ plane as shown by the faint-shaded region of Figure 1.

The time needed in order to *find* tens of supernovae is significantly larger. For example, at a 10-meter telescope about 15ϵ minutes would be needed to find a supernova at $z \approx 0.5$, using a wide-field camera such as the four-CCD mosaics currently being commissioned at several observatories. Using the 2.4-meter Hubble Space Telescope as suggested by Colgate (1979) to study high-redshift SN would not significantly diminish the length of exposures needed for SN at redshifts $z \lesssim 1$ (see Nelson, Mast, & Faber 1985 for Keck-HST comparisons).

Based on a 1-hour spectrum of a supernova at $z = 0.425$ observed at a 3.6-meter telescope (Perlmutter *et al.* 1994b), we estimate that 10 hours of 10-meter telescope time are required to obtain a spectrum of a supernova at $z = 1$, and only 15 minutes

for supernovae at $z = 0.5$. For the first set of high redshift SNe, these spectra would be necessary in addition to color photometry to check identification and evolution. If these spectra show no surprises, it may be possible to spot check the subsequent SN spectra and use multicolor lightcurves instead.

In this estimate of the observation time required, we have implicitly included the K -correction by moving to a longer-wavelength band for the higher redshift measurements. An important calibration step in the actual experimental protocol for this measurement will be the careful determination of the K -correction for each SN studied. Currently available spectra of nearby SNe allow traditional K -correction estimates (e.g., corrections for light emitted in the B band at high redshifts to the light observed in the B band) to be made with reasonable accuracy (< 0.05 mag) out to redshifts of order $z \approx 0.2$, within less than 20 days (SN rest frame) of maximum light (Hamuy et al 1993). A generalization of the K -correction that corrects for light emitted in the B band, for example, at high redshifts, but observed in the R band can be calculated with this same accuracy for objects out to at least $z = 0.6$ (Kim, Goobar, & Perlmutter 1995). However, for the most accurate corrections, particularly at high redshifts, it will be important to make further well-calibrated observations of a number of newly discovered nearby SN Ia spectra and lightcurves, to ensure that any supernova-to-supernova differences are sampled. In particular, it may be useful to observe nearby SNe Ia with a range of filters specifically designed to match “blueshifted” I or R standard filters for a sample of redshifts (e.g., for $z = 0.3, 0.4, 0.6, 0.7$; the current data in B and V may serve for “blueshifted” R at $z \approx 0.5$ and 0.2 , or I at $z \approx 0.8$ and 0.5). This would allow an accurate K -correction interpolation table to be constructed. Note that this K -correction work requires a well-calibrated data set, since any wavelength-dependent error in the K -corrections could mimic redshift-dependent changes in magnitude, and hence confound the measurements of Ω_M and Ω_Λ .

In practice, actual telescope observing time is, of course, always significantly longer than the theoretical predicted time. These time estimates are intended to convey the scale of this observing program; it is an ambitious but practicable program.

6. Discussion

As with the other methods for determining the cosmological constant discussed in the introduction, this approach depends on results from an entire research program. More nearby SNe Ia must be discovered and studied, as expected from a few projects (e.g. Hamuy et al 1993b; Muller et al 1992). This will make it possible to test and refine the criteria used to distinguish “normal” un-extincted SNe Ia, to further develop lightcurve decay-time/shape calibration, and to determine the true intrinsic magnitude distribution. Distant SNe Ia must also be discovered before maximum light on a regular basis (e.g., Perlmutter *et al.* 1994a, 1994b), and the observational effort necessary to study them as outlined in this paper will not be trivial. Both the nearby and distant SNe Ia will contribute to the tests for evolution. Finally, careful photometric and spectral work will still be needed to ensure that the uncertainty in the K -corrections is negligible compared to the other sources of error. Given that research programs are already underway in all of these domains, this approach to the measurement of Λ and Ω_M may soon be feasible.

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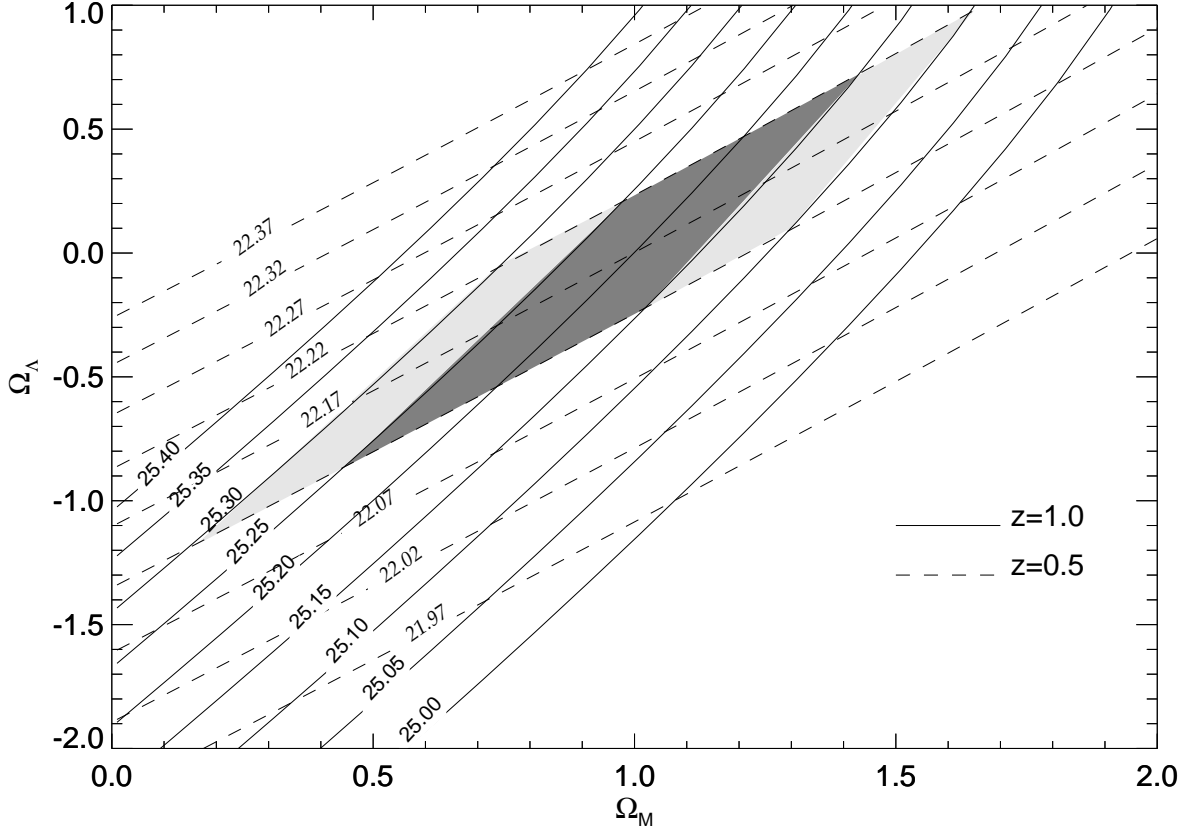


Fig. 1.— Contours of constant apparent magnitude (R-band) predicted for an example standard candle with absolute magnitude (B-band) $M_B = -18.86 + 5 \log(H_0/75)$. The dashed lines show the predicted apparent magnitude, including K -corrections, for a standard candle at $z = 0.5$ and the dashed lines are for $z = 1$. The dark shaded region shows the “allowed” region of Ω_Λ -versus- Ω_M parameter space if an apparent magnitude of $m_R = 22.17 \pm 0.05$ were measured at $z = 0.5$ and $m_R = 25.20 \pm 0.05$ were measured at $z = 1$. Adding the faint shaded region implies a 0.1 magnitude uncertainty for supernovae at $z = 1$.

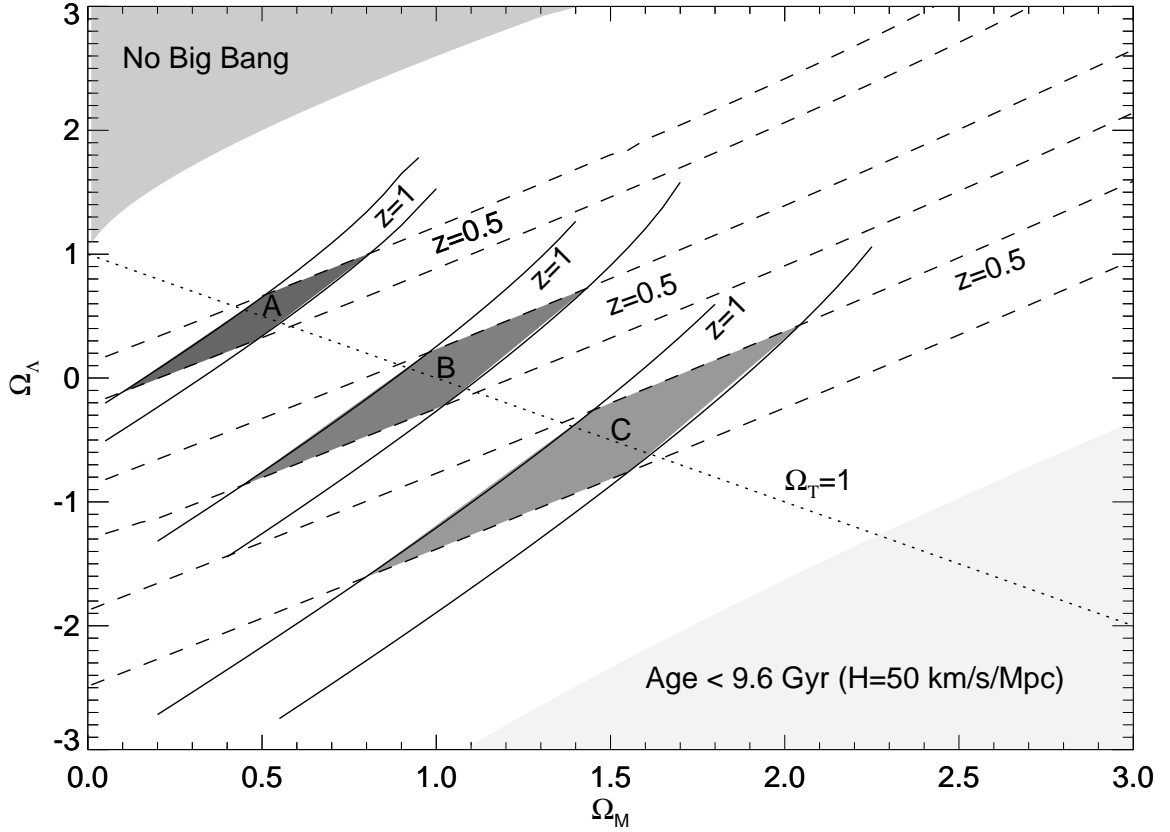


Fig. 2.— The map of parameter space for Ω_Λ and Ω_M . The top and bottom shaded areas are ruled out by observations (see text). The solid lines show the enclosed band that a 0.05 mag measurement a standard candle at $z = 1$ would imply for three different universes. Similarly, the dashed lines correspond to the same standard candle at $z = 0.5$. The regions A, B and C give the allowed parameter space for the cases when the parameters are $(\Omega_\Lambda=0.5, \Omega_M=0.5)$ for A, $(\Omega_\Lambda=0.0, \Omega_M=1.0)$ for B and $(\Omega_\Lambda=-0.5, \Omega_M=1.5)$ for C.

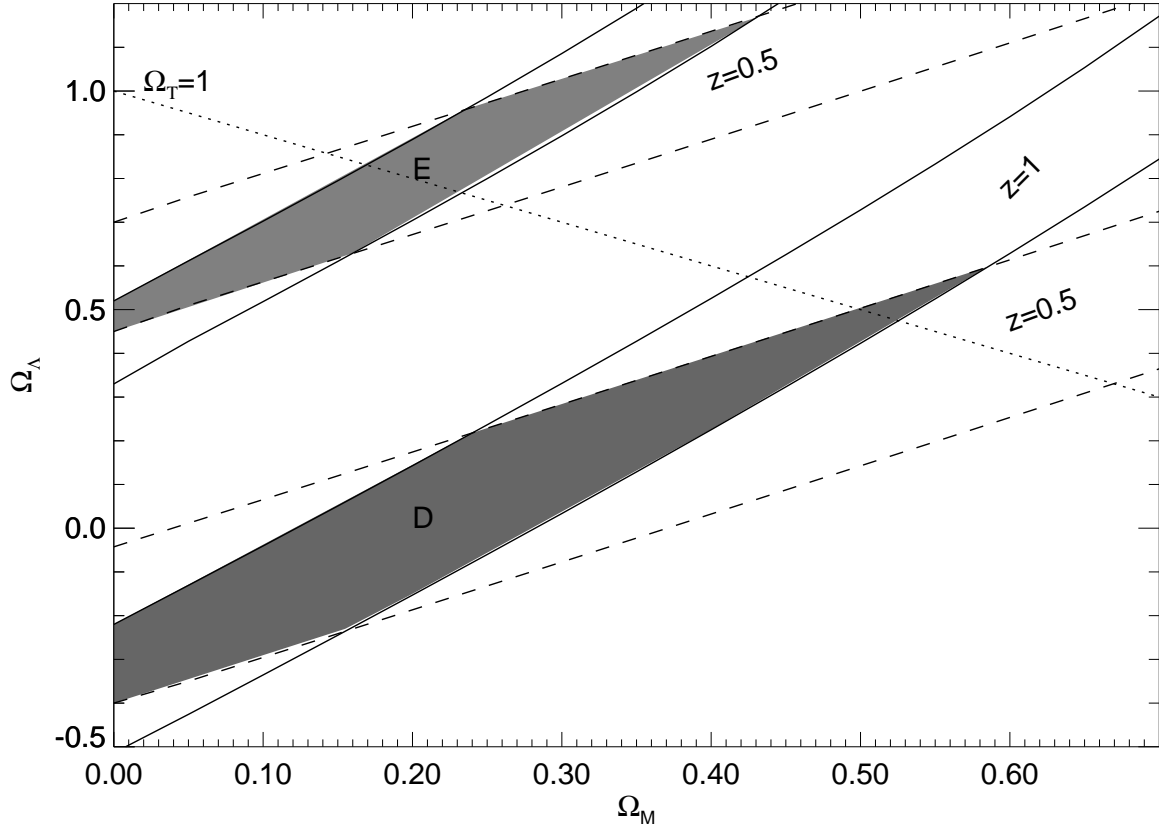


Fig. 3.— The map of allowed parameter space for Ω_Λ and Ω_M . The region D corresponds to $\Omega_\Lambda=0$ and $\Omega_M=0.2$. E corresponds to $\Omega_\Lambda=0.8$ and $\Omega_M=0.2$

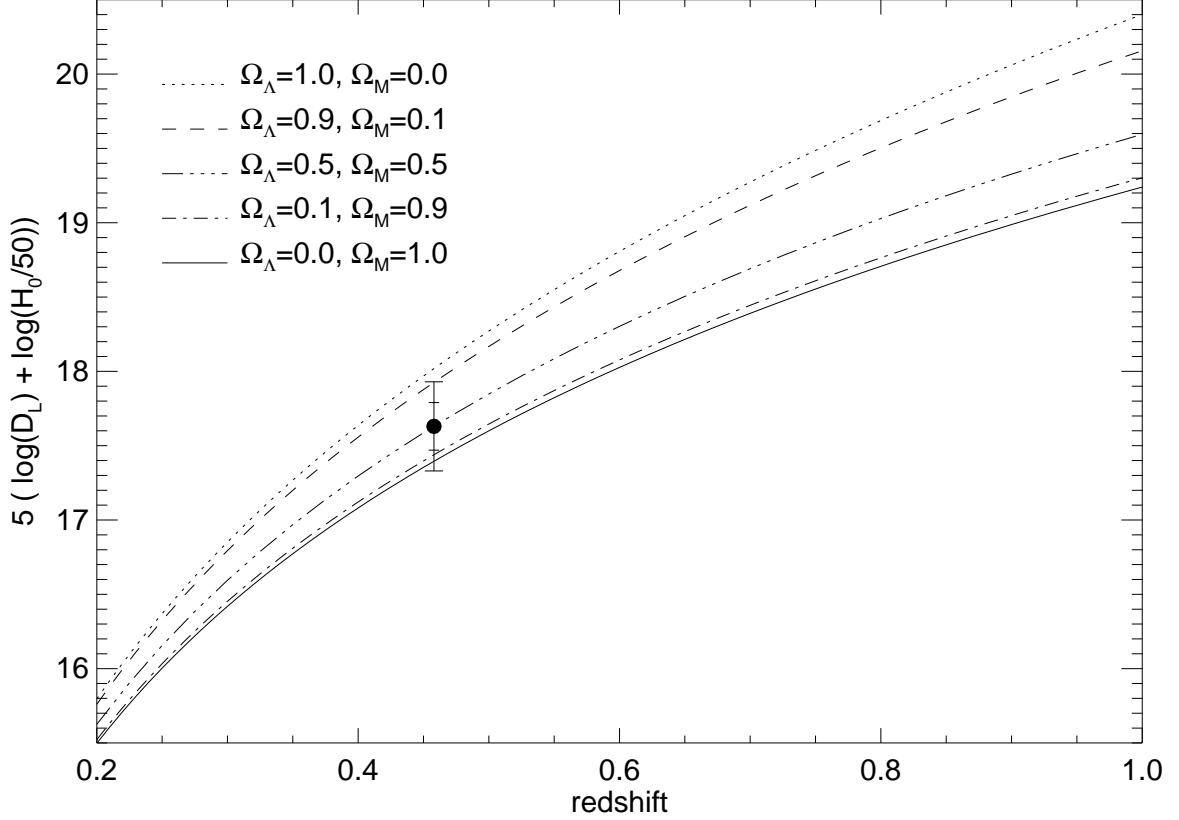


Fig. 4.— Luminosity distance as a function of redshift for various values of Ω_M and Ω_Λ in a flat universe ($\Omega_M + \Omega_\Lambda = 1$). The filled circle corresponds to $(m - M - K - 25)$ for SN1992bi (Perlmutter *et al.* 1994a), where the smaller error bar is due to the photometry measurement error, $\sigma^{\text{photometry}} \approx 0.15$, and the larger error bar includes a 0.25 magnitudes intrinsic dispersion for type Ia SNe.